

Grid-generated turbulence in dilute polymer solutions

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Measurements were made of some of the properties of grid-generated turbulence for several concentrations of a drag-reducing polymer additive in water. Reference measurements of the grid pressure drops, streamwise intensities and one-dimensional spectra in pure water agreed with previous measurements obtained in Newtonian fluids. Corresponding results for the polymer solutions showed that the grid pressure drops were generally lowered, the turbulence intensity levels were increased and the one-dimensional energy spectra were unchanged compared to the results in water. A dimensionless rate of decay correlation was introduced which removed the dependence of the rate of decay on the initial conditions of grid-generated turbulence for Newtonian fluids: the polymer solution decay results did not follow this correlation, indicating that the decay process is different in these fluids. The one-dimensional energy spectra of the turbulence in the polymer solutions were of the same shape as those in Newtonian fluids, and were normalized with modified Kolmogoroff variables using an effective viscosity (Lumley 1964) and the viscous dissipation in order to provide a quantitative comparison to Newtonian spectra. All of the normalized spectra in the polymer solutions collapsed to a single curve which coincided with the normalized Newtonian spectral curve. It was also found that the rate of decay was less than the viscous dissipation in the polymer solutions, in accordance with recent theoretical predictions.

1. Introduction

The reduction of pressure losses and drag force in the turbulent shear flows of liquids caused by the addition of small amounts of certain polymeric materials has received considerable attention. Most studies have dealt with pressure drop-flow rate measurements in pipes, from which correlations predicting the onset of drag reduction have been obtained (see e.g. Virk & Merrill 1969). However, the basic reasons for the occurrence of drag reduction do not appear to be completely understood. Crucial to a more complete understanding of the phenomenon are measurements and analyses of the statistical properties of turbulent velocity fields in drag-reducing liquids. Our experiments are concerned with the effects of a drag-reducing polymer additive on some of the properties of a grid-generated turbulent flow.

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There have been previous measurements of some turbulence properties in drag-reducing fluids, both in grid and shear flows. Fabula (1966) measured the energy spectra of the streamwise turbulent velocity in a grid-generated flow of solutions of polyethylene oxide in water. The turbulence was produced by towing a grid in a quiescent tank of fluid, and the spectral measurements were obtained with a conical hot-film anemometer probe. Fabula noted that the spectra exhibited higher energy at large wave-numbers in freshly prepared polymer solutions compared to pure water, and called this effect 'signal raggedness'. The spectra were believed to be anomalous because the dissipation of turbulent kinetic energy calculated from the solution viscosity and the second moment of the spectra, was five times the dissipation given by Kolmogoroff's relation for the inertial subrange for Newtonian fluids. The spurious high frequency signals were thought to arise from polymer agglomerates striking the probe (see also Lumley 1967, 1969). In aged solutions, the anomalous spectral form was not observed, and the spectra were qualitatively the same as in pure water. Barnard & Sellin (1969), in a flow visualization study of the injection of polyethylene oxide solutions into a grid-generated turbulent field, observed a qualitative reduction in the fine scale structure of the turbulence compared to that in water. Measurements of photographs of the dye tracer jets showed reduced spreading angles, and were interpreted as reduced turbulence intensity in the polymer solutions. Virk *et al.* (1967) obtained one-dimensional spectra in the turbulent pipe flow of a polyethylene oxide-water solution. In those measurements a cylindrical probe was used, although it was noted that in freshly prepared solutions this type of probe exhibited anomalous calibration effects (Smith *et al.* 1967). Friehe & Schwarz (1969), in an anemometer probe calibration study in another drag-reducing fluid, found that cylindrical and conical probes gave different spectral forms in grid-generated turbulence. The cylindrical probe indicated a power law dependence of the spectrum on wave-number at high wave-numbers, whereas the conical probe gave the exponential-like roll-off as found in Newtonian fluids. On the pipe axis, Virk *et al.* found the spectrum to be different from that measured in water. At large wave-numbers, the spectrum exhibited a power law dependence, which is perhaps due to the type of probe that was used. The measurements also showed an increased intensity level of 40% over that in water, while the dissipation, calculated from the isotropic spectral relation and the solution viscosity (reported to be Newtonian), was 40% lower. Lumley (1969), however, has stated that the resolution of the probe used in those measurements was not adequate in the dissipation range of wave-numbers. Similar measurements near the pipe wall showed that the intensity, spectral shape and dissipation were the same as in pure water. Johnson & Barchi (1968) measured the relative effects of the injection of polyethylene oxide into a turbulent boundary layer using a conical hot-film probe. It was found that the intensity level of the turbulence near the wall increased with injection, and the spectrum showed more energy at low wave-numbers and less at high wave-numbers compared to that in water. Spangler (1969) obtained measurements similar to those of Virk *et al.* (1967) in a pipe, but with a different polymer (an ionic copolymer of polyacrylamide and polyacrylic acid), and used an impact tube from which instantaneous velocity

measurements were inferred. The intensity level at the centre-line of the pipe was increased up to three times that in water, the effect decreasing with increasing mean velocity. Spectral shapes were found to be the same as in water with this type of probe. In summary, it appears that the pipe-flow measurements indicate a higher intensity level at the centre-line, but the spectral data differ, depending on the type of probes and polymer solutions that were used.

Some theoretical studies of turbulence in non-Newtonian fluids have been reported. Since polymer solutions do exhibit non-Newtonian effects, the results of those theories may be relevant to the understanding of drag reduction. Lumley (1964) examined the dissipation of turbulent energy in an isotropic, homogeneous turbulence for Reiner–Rivlin fluids and concluded that the spectrum will be the same as in a Newtonian fluid, except characterized by an effective viscosity dependent on the mean-square strain rate of the turbulence. Further, the energy containing range of wave-numbers, in which viscous forces are not important, was found to be unchanged compared to the Newtonian case. The Reiner–Rivlin constitutive equation is of the differential type, and hence the stress tensor depends on the history of the deformation gradient $\mathbf{F}^t(\tau)$ in a very special way, that is, it depends only on $\mathbf{F}^t(\tau)$. The model predicts a shear-dependent viscosity, but the predicted normal stresses in viscometric flows have not been observed in any fluid experimentally studied to date. Despite the fact that the Reiner–Rivlin constitutive equation used by Lumley (1964) in his analysis is deficient, the concept of an effective viscosity can be useful for fluids which require a more general constitutive representation.

Singh (1966) theoretically studied isotropic, homogeneous turbulence in a second-order fluid by modifying Heissenberg's energy spectrum analysis. This fluid exhibits a constant shear viscosity, and requires two additional constants for the normal stresses. The primary result of the analysis was the prediction of a power-law spectrum at high wave-numbers, with a smaller exponent, and more energy, than in the Newtonian case at these wave-numbers. Chow & Saibel (1967) analysed the decay of isotropic, homogeneous turbulence for an Oldroyd fluid, which is characterized by five material constants, to represent the viscous, relaxation and normal stress effects. The rate of decay of the turbulent kinetic energy was found to be equal to the viscous dissipation, plus two additional terms containing the non-Newtonian parameters. Whether the rate of decay was increased or decreased compared to that in Newtonian fluids depended on the relative magnitude of the parameters, relaxation effects increasing, and normal stresses decreasing, the rate of decay. Singh's analysis also showed that the energy budget was altered in second-order fluid turbulence: the rate of decay was found to be equal to the viscous dissipation minus a term due to normal stresses.

Our experiments reported here consist of measurements of grid pressure drops, the decay of the streamwise intensities and the one-dimensional energy spectra for various concentrations of polyacrylamide-water solutions in a grid-generated turbulent field. The data were compared to similar data obtained in the pure solvent and also analysed in accordance with the several previous theoretical results about turbulence in non-Newtonian fluids.

2. Experimental details

The experiments were performed in a recirculating fluid tunnel of approximately 450 gallons capacity fitted with a 6 in. square \times 60 in. long test section. Screens and a contraction were placed upstream of the test section to provide a uniform, low turbulence intensity level flow in the section (Gibson & Schwarz 1963). A biplane grid of round rods with a mesh (M) of $\frac{1}{2}$ in. and diameter (d) of $\frac{3}{32}$ in. was used to produce the grid-generated turbulence. Provision was not made in the tunnel to compensate for the axial increase of the mean velocity due to boundary-layer growth on the test section walls, nor to improve the isotropy of the turbulence by the method used by Comte-Bellot & Corrsin (1966). Static pressure taps were placed 2 in. upstream and 27.5 in. downstream of the grid to measure the grid pressure drop. Inverted U-tube manometers were used to measure the grid pressure drops and the pressure differences from Pitot-static tubes. In this study, the use of Pitot-static tubes to measure velocity in the test section without a grid was verified experimentally by comparison to the velocity of hydrogen bubble tracers (Friehe & Schwarz 1969). It was assumed that the Pitot-static tube also indicated the mean velocity in the grid-generated field without a significant error.

The water soluble polymer used was Dowell J-100, an ionic polyacrylamide which exhibits drag reduction in pipes (Ripkin & Pilch 1964). Deionized water was used to prepare nominal concentrations of 10, 20, 30, 100 and 300 ppm (parts per million by weight) based on a tunnel volume of 450 gallons. Concentrated solutions containing all of the polymer were prepared in a barrel and transferred to the tunnel (without a pump to avoid degradation) for final mixing and dilution.

Rheological data were obtained for the 100 and 300 ppm solutions, and the dependence of the viscosity μ on the strain rate κ for the 300 ppm solution is shown in figure 1. These data were obtained with both a Weissenberg rheogoniometer and a variable pressure capillary viscometer to span the strain rate range of approximately 10^{-1} to 10^4 sec $^{-1}$. Two sets of data are shown on figure 1, for the freshly prepared solution and the same solution after several hours use in the tunnel. Mechanical degradation of the polymer solution in the tunnel lowers the viscosity at low rates of strain and the effect of this on the turbulence measurements is discussed in a later section. Normal stresses, measurable on the rheogoniometer, were not observed for the solutions used in this work. A more complete rheological study including normal-stress measurements on a similar water-polymer system for a wide range of concentrations has been reported by Bruce & Schwarz (1969). The densities of the solutions were taken to be that of pure water.

Turbulent velocity data were obtained with a linearized, constant temperature anemometer (Thermo-Systems, Inc., units 1010A and 1005B) using a quartz-coated conical hot-film probe (also of Thermo-Systems design). In most turbulence measurements in air, the bare hot-wire is the basic probe configuration used. For use in liquids, quartz-coated cylindrical hot-film probes (similar in shape to hot-wires) are available, but the following anomalous effects have been observed using those probes in drag reducing fluids: greatly reduced heat transfer coefficients (compared to those in pure water); ranges of velocity for which the

heat transfer coefficient is constant; and, for concentrated solutions, opposite dependence, compared to water, of the heat transfer on the angle of yaw of the flow direction to the cylinder axis (Friehe & Schwarz 1969). These effects, and the above-mentioned different spectral forms obtained with cylindrical and conical probes, seem to indicate that the interpretation of turbulence measurements with cylindrical probes is open to question until comparisons can be made

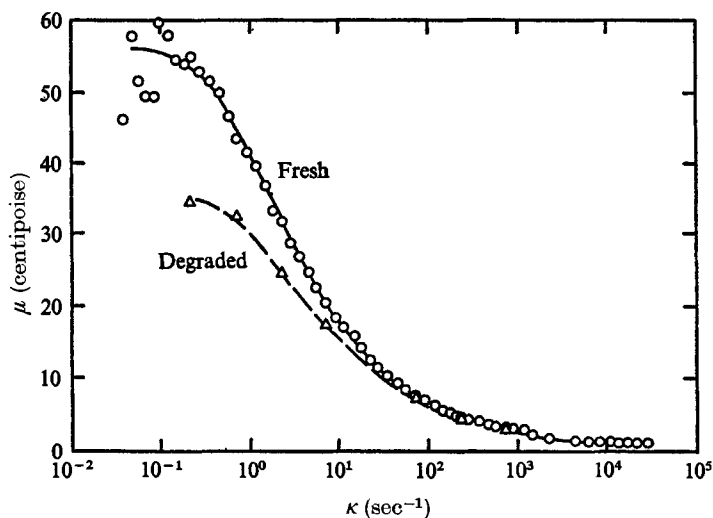


FIGURE 1. Strain rate dependent viscosity of 300 ppm J-100: O, fresh solution; Δ , degraded solution.

with other instantaneous velocity-measuring devices, e.g. the laser velocimeter. The conical probe used in the J-100 solutions exhibited heat transfer calibrations (of anemometer voltage *versus* mean velocity) similar to those obtained in pure water, and it was assumed that the probe correctly measured the instantaneous velocity. Other equipment used for the turbulence measurements were a Ballantine Model 320 True Root-Mean-Square voltmeter, a Hewlett-Packard 302A wave analyser, and an integrating digital voltmeter.

3. Results

(i) Water

Reference measurements of the grid pressure drop, axial variation of the stream-wise intensity and one-dimensional spectra were obtained in pure water for comparison to similar data obtained in the polymer solutions. The water data were also compared to existing grid turbulence data obtained primarily in air.

(a) *Grid pressure drops.* The variation of the grid pressure drop Δp , expressed as the pressure coefficient $C_p = 2\Delta p/\rho\bar{U}^2$, where ρ = density and \bar{U} = mean velocity, with the grid Reynolds number $Re_M = \bar{U}M/\nu$ (ν = kinematic viscosity), as shown in figure 2. The magnitudes of the C_p values are approximately the same as the value of $C_p = 0.4$ reported by Corrsin (1963) for a similar grid at an unspecified Reynolds number. By comparison with the intensity measurements

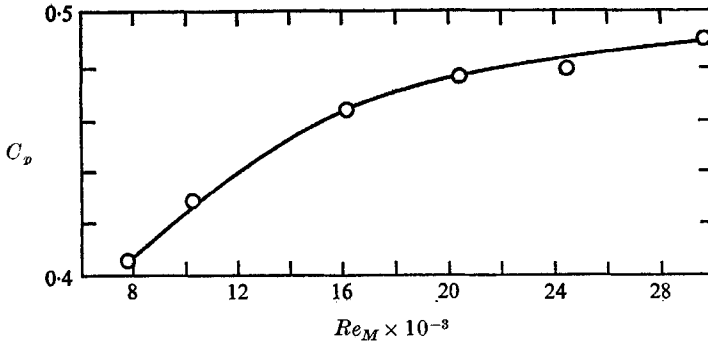


FIGURE 2. Grid pressure drop coefficient *versus* grid Reynolds number for water.

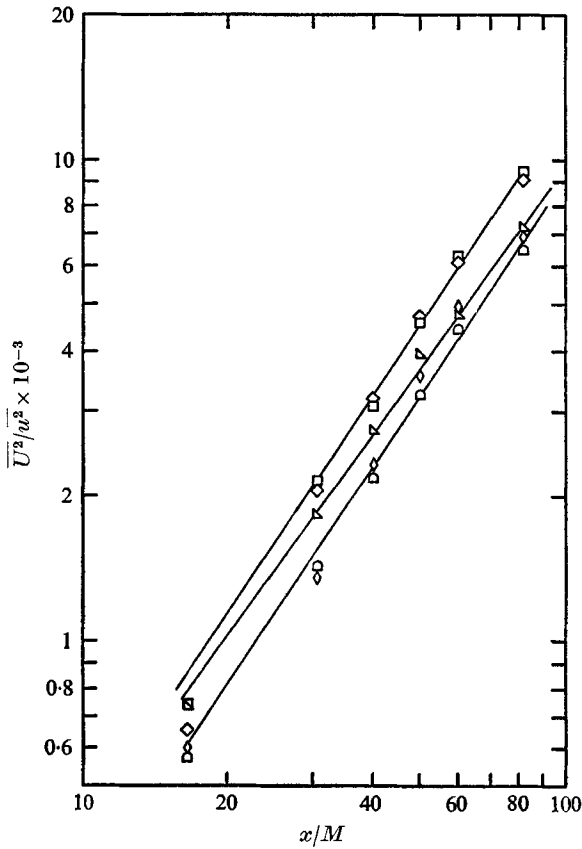


FIGURE 3. Variation of \bar{U}^2/u^2 with x/M for water for various Re_M .

$Re_M \times 10^{-3}$	n_1
□ 9.55	1.47
◇ 10.4	1.47
∧ 15.8	1.37
△ 19.7	1.37
◇ 23.7	1.47

shown in figure 3, the streamwise intensity $\overline{u^2}$ increased with \overline{U} and hence C_p , but the data were not sufficiently accurate to confirm that $\overline{u^2} \propto C_p$ as predicted by Batchelor (1960).

(b) *Intensity and rate of decay.* Also shown in figure 3 is the variation of $\overline{U^2}/\overline{u^2}$ with the dimensionless axial distance from the grid, x/M , for the range of grid Reynolds numbers of 9.55×10^3 to 23.7×10^3 . The data are described by the well-known power law formula,

$$\frac{\overline{U^2}}{\overline{u^2}} \propto \left(\frac{x}{M} - \frac{x_1}{M} \right)^{n_1},$$

where $n_1 = 1.37$ to 1.47 and the virtual origin, x_1/M , was taken to be zero, since the scatter of the data did not justify estimating its small value. Measurements of $\overline{v^2}$ and $\overline{w^2}$, the intensities of the cross-stream components, were not obtained, but were assumed to be given by $\overline{v^2} = \overline{w^2} = 0.83\overline{u^2}$ after Comte-Bellot & Corrsin (1966).

The rate of decay of the intensity is given by

$$\epsilon = -\frac{1}{2} \frac{d}{dt} (\overline{u^2} + \overline{v^2} + \overline{w^2}) = -\frac{1}{2} \frac{d}{dt} (\overline{u^2} + 2\overline{v^2}),$$

which can be written as

$$\epsilon = -\frac{1}{2} \frac{\overline{U^3}}{M} \frac{d}{d(x/M)} \left(\frac{\overline{u^2}}{\overline{U^2}} + 2 \frac{\overline{v^2}}{\overline{U^2}} \right), \quad (1)$$

with the introduction of M and \overline{U} as characteristic length and velocity scales, and using the space-time transformation $dx = \overline{U} dt$. For comparison with the polymer solution results, it is of interest to introduce a normalized rate of decay which is independent of the variable 'initial conditions' of the turbulent field produced by different types of grids and dynamical conditions (Re_M). Fabula (1966) formed the dimensionless quantity $\epsilon M/\overline{U^3}$, and found that the decay data for biplane grids of square rods (Comte-Bellot & Corrsin) could be accurately correlated by plotting $\epsilon M/\overline{U^3}$ versus x/M over the range

$$1.7 \times 10^4 \leq Re_M \leq 13.5 \times 10^4$$

($\pm 7\%$ variation in $\epsilon M/\overline{U^3}$ at a given x/M). However, this correlation removes only the dependence of $\epsilon M/\overline{U^3}$ on Re_M ; other types of grids obtain different correlations each of which is independent of Re_M . For isotropic turbulence, Batchelor (1960) derived the interesting result $\lambda^2 = 10\nu t/n_1$, which relates the Taylor microscale λ to the power law decay formula $\overline{u^2} \propto t^{-n_1}$. Only the parameter n_1 of the power law appears in the equation, which suggests using

$$\tilde{\epsilon} = \epsilon M / (\overline{u^2} + 2\overline{v^2}) \overline{U}$$

for the dimensionless rate of decay, since $(\lambda^2)^{-1}$ is proportional to $1/\overline{u^2} d\overline{u^2}/dt$ for isotropic turbulence. In fact, for the decay of anisotropic turbulence, where

$$\frac{\overline{U^2}}{\overline{u^2}} = A_1 \left(\frac{x}{M} - \frac{x_1}{M} \right)^{n_1} \quad \text{and} \quad \frac{\overline{U^2}}{\overline{v^2}} = A_2 \left(\frac{x}{M} - \frac{x_2}{M} \right)^{n_2},$$

and assuming $x_1/M = x_2/M$ and $n_1 = n_2$ (approximately shown by the results of Comte-Bellot & Corrsin), (1) yields

$$\bar{\epsilon} = \frac{n}{2} \left(\frac{x}{M} - \frac{x_0}{M} \right)^{-1}, \quad (2)$$

where
$$n = \frac{n_1 + n_2}{2} \quad \text{and} \quad \frac{x_0}{M} = \frac{\frac{x_1}{M} + \frac{x_2}{M}}{2}.$$

The quantity $\bar{\epsilon}$ is related to the dimensionless inverse of the Taylor microscale by $\bar{\epsilon} = 5\nu M/\lambda^2 \bar{U}$ and depends only on n and x_0/M . Previous decay data tabulated by Comte-Bellot & Corrsin show $1.0 \leq n \leq 1.37$ and $0 \leq x_0/M \leq 9$; their accurate measurements in air for biplane grids of round and square rods gave

$$1.17 \leq n \leq 1.30 \quad (\pm 0.02) \quad \text{and} \quad 1.5 \leq x_0/M \leq 3.5 \quad (\pm 1.0).$$

With the latter data, $\bar{\epsilon}$ versus $(x/M)^{-1}$ is shown in figure 4 and compared to the present results obtained in water for the x/M range 20 to 400, the maximum for which the power law formulas are expected to be valid. Some difference in the correlations between the data in water and air is shown, due to lack of precision of the results in water, but the correlation is adequate for comparison to the decay data obtained in the polymer solutions. The decay data of Comte-Bellot & Corrsin give a $\pm 5\%$ variation in $\bar{\epsilon}$ at a given x/M , whereas the intensity level varied about a factor of two over the range $1.7 \times 10^4 \leq Re_M \leq 13.5 \times 10^4$ for the two types of grids. The present correlation, based on the data of Comte-Bellot & Corrsin, gives a slight improvement in accuracy over that of Fabula, and is valid for both round and square rod grids. Expansion of (2) for $x/M > x_0/M$ gives

$$\bar{\epsilon} \doteq \frac{n}{2} \left(\frac{x}{M} \right)^{-1} + \frac{n}{2} \left(\frac{x_0}{M} \right) \left(\frac{x}{M} \right)^{-2}.$$

The variation of the coefficient, $\frac{1}{2}n$, of the first term is $\pm 5\%$. Examination of the data of Comte-Bellot & Corrsin for the coefficient of the second term indicates that it is approximately constant: large values of n are associated with small x_0/M values, and vice versa, such that $\frac{1}{2}n(x_0/M) \cong 1 - 2.4 (\pm 0.5)$ (with the exception of the choice of $x_0/M = 0$). Comte-Bellot & Corrsin also obtained decay data for a grid composed of disks (which gave intensity levels approximately twice those of the round and square rod grids) for which $1.32 \leq n \leq 1.38$ and $2 \leq x_0/M \leq 6$: those data result in a $\pm 10\%$ variation in $\bar{\epsilon}$ when included with the round and square rod data. The correlation of $\bar{\epsilon}$ given by (2) appears adequately to remove ($\pm 10\%$) the dependence of the rate of decay on different intensity levels produced by different types of grids and grid Reynolds numbers. It also appears possible to determine accurately ($\pm 5\%$) the rate of decay (and the Taylor microscale) for round and square rod grids from the correlation shown in figure 4 at a given x/M by a measurement of $\overline{u^2} + \overline{2v^2}$ at that point, an improvement over the usual lengthy experimental determination of the power law parameters.

(c) *Spectra.* Four one-dimensional spectra, $\phi(k)$, where k is the wave-number, and

$$\overline{u^2} = \int_0^\infty \phi(k) dk,$$

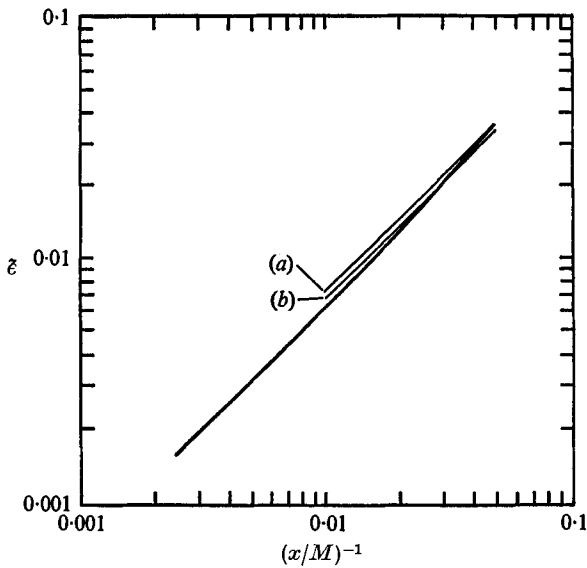


FIGURE 4. Dimensionless decay of grid-generated turbulence for Newtonian fluids. —, data of Comte-Bellot & Corrsin (1966). (a) —, present data, $n_1 = 1.47$. (b) —, present data, $n_1 = 1.37$.

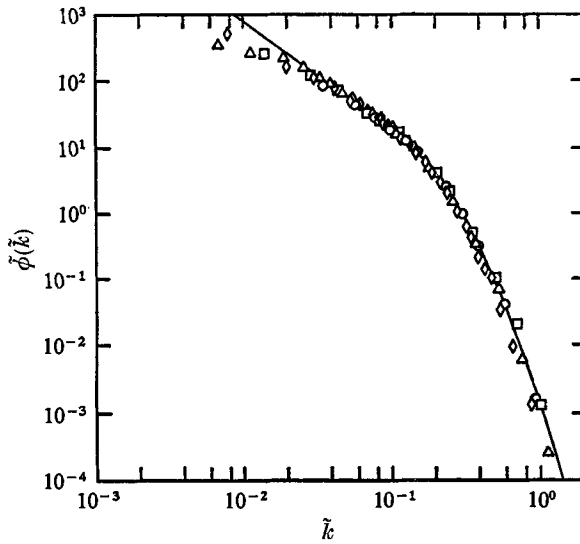


FIGURE 5. Normalized one-dimensional energy spectra in water, compared to curve (—) of Gibson & Schwarz (1963).

$Re_M \times 10^{-3}$	x/M	Probe
○ 7.6	31.75	0.001 in. diam. cylinder
△ 14.2	51.75	0.001 in. diam. cylinder
□ 14.2	31.75	0.001 in. diam. cylinder
◇ 14.2	31.75	Conical probe

of the streamwise fluctuating component u were obtained, three with 0.001 in. diameter cylindrical hot-film probes, and one with the conical probe. The spectra were made dimensionless, $\check{\phi}(\check{k})$, with the rate of decay (with the assumption about anisotropy as mentioned above), and the kinematic viscosity by $\check{\phi}(\check{k}) = \phi(k)/\nu^2 k_K$ and $\check{k} = k/k_K$, where $k_K = (\epsilon/\nu^3)^{1/4}$, the Kolmogoroff wave-number. The results are shown in figure 5, and compared to the compilation of previous spectral results obtained for grid-generated turbulence in air and water, and one spectrum in the ocean (Gibson & Schwarz 1963). The present results agree with the curve of Gibson & Schwarz in the limited $\check{k}^{-5/3}$ region, but are slightly lower at high wave-numbers ($\check{k} \gtrsim 10^{-1}$), presumably due to frequency response limitations of the quartz-coated probes. The dimensionless second moment of the spectra,

$$\int_0^\infty \check{k}^2 \check{\phi}(\check{k}) d\check{k},$$

obtained by graphical integration, was 0.055 for the average of the four spectra, as compared to the theoretical value of $\frac{1}{16}$ (0.0667). The conical probe data showed slightly more attenuation at high wave-numbers than the data obtained with the cylindrical probes, but the conical probe results are believed precise enough for comparison with the spectral data obtained in the polymer solutions.

(ii) Polymer solutions

(a) *Grid pressure drops.* The variation of the grid pressure-drops with mean velocity, for the polymer solutions (except 300 ppm), are shown in figures 6(a)–(d); and the data at high velocities generally show reduced values of C_p of the order of 10% compared to those in pure water, which are another form of drag reduction. (It should be noted here that recently it has been found that the use of wall pressure taps with certain non-Newtonian fluids has indicated incorrect pressure measurements (Tanner & Pipkin 1969); but such an effect is not expected here, since the static pressure differences were measured along the straight test section wall, and presumably such errors would subtract out). The dependence of C_p on \bar{U} for the polymer solutions differs from that in the case of water. Further, the use of a constant viscosity to form a grid Reynolds number does not appear to result in a matching to the water results shown in figure 2. In figure 6(a) results for both fresh and degraded 10 ppm J-100 are shown; and, with degradation, the C_p values approach those of pure water. Similar measurements for the 300 ppm solution are given in figure 6(e), and the data are notably different from the other solutions and water, with C_p values 25 to 45% higher than those in water. These pressure measurements indicate that the nature of the flow around the grid rods is sufficiently changed in the polymer solutions, that the size of a pressure-drop (a gross measure of the ‘initial conditions’ of the grid-generated turbulent field) is different from one in pure water.

(b) *Intensity and rate of decay.* The measurements of $\bar{U}^2/\overline{u^2}$ versus x/M at various mean velocities for the polymer solutions are shown in figures 7(a)–(e). In general, the results show increased intensity levels over those in pure water and also a different dependence on x/M . The precision of the data is not good, due

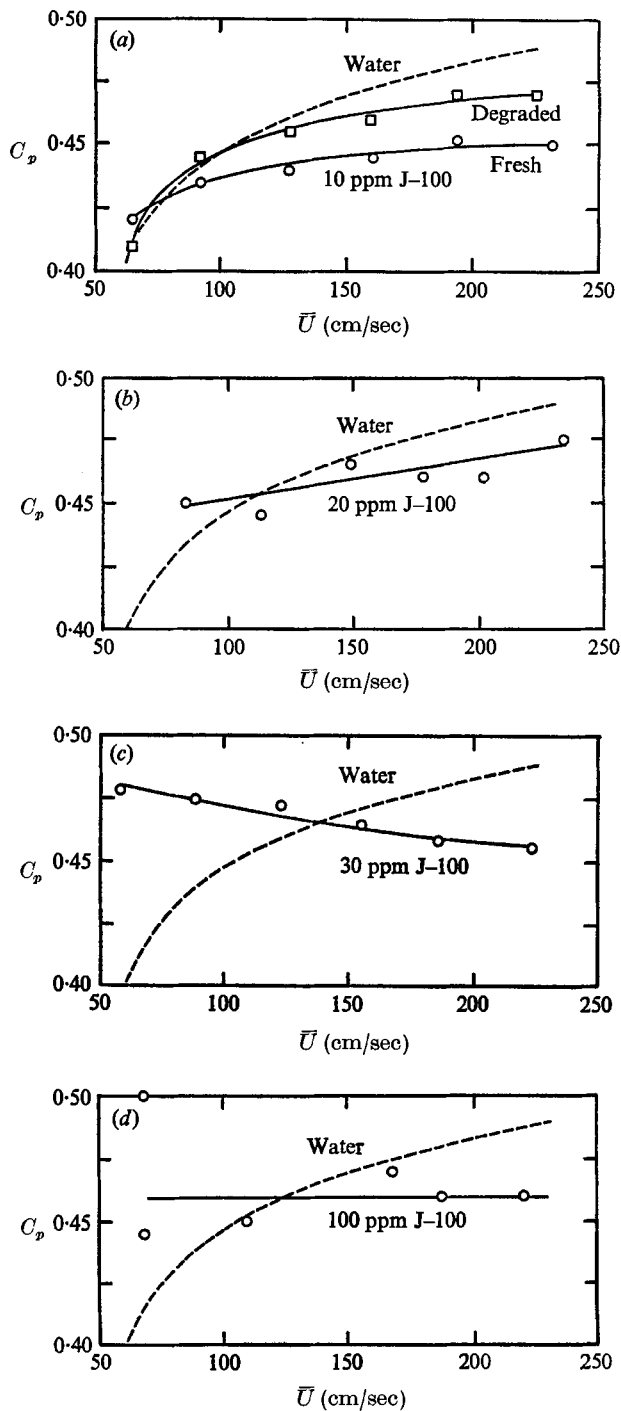


FIGURE 6. Grid pressure drop coefficient *versus* mean velocity for polymer solutions, compared to water data (---): (a) 10 ppm J-100, fresh and degraded, (b) 20 ppm J-100, (c) 30 ppm J-100, (d) 100 ppm J-100, (e) 300 ppm J-100.

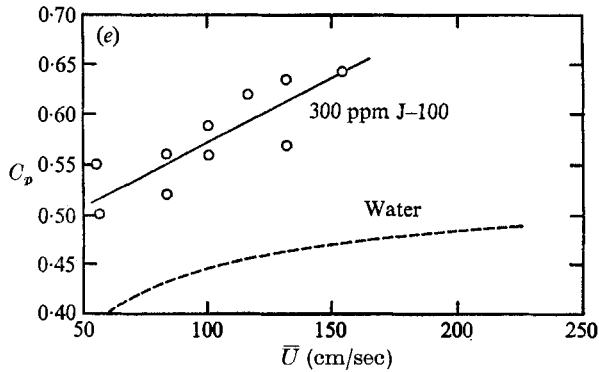


FIGURE 6(e). For legend see p. 183.

largely to degradation of the polymer solutions with time of use in the tunnel. As the solutions were pumped for several hours, the intensity levels gradually decreased. For some runs, it appears that the power law formula, which describes intensity data in Newtonian fluids, also holds for the polymer solutions with larger values of A_1 and smaller values of n_1 . From the data, it appears that the intensity level increases with increasing mean velocity, as in water, and also with increasing polymer concentration; but the exact forms of the dependences are not clear, due to the excessive scatter of the data.

From the intensity data, it was found that the rates of decay ϵ in the polymer solutions were higher than those in pure water; e.g. at 30 ppm and 155 cm/sec, the ratio of ϵ in the solution to ϵ in water increased linearly from 1 at $x/M = 16.5$ to 8 at $x/M = 100$. This was due to the much higher intensity levels in the polymer solution, even though the power law exponents were lower. The comparisons of the rates of decay in the polymer solutions with those in water, on the dimensionless basis introduced earlier, are shown in figure 8 for the 30 ppm solution. There are significant differences between the dimensionless decay correlation found to hold in Newtonian fluids and those in the polymer solutions, which show that the decay process is altered by the addition of polymer.

(c) *Spectra.* One-dimensional energy spectra were obtained in the 30, 100 and 300 ppm solutions with the conical hot-film probe. Qualitatively, all of the spectra were of the same shape as those in Newtonian fluids. However, quantitative comparison is required in order to definitely determine whether the spectra are identical to those in the pure solvent. It is possible that the spectra, normalized in some manner, may not coincide with the Newtonian universal spectral curve shown in figure 5, although they are of the same shape. The following describes the spectral normalization used to correlate the present data. Because the more concentrated solutions exhibited shear dependent viscosities, Lumley's concept of an effective viscosity was used: the effective viscosities ν_e were calculated from the spectral relation

$$\epsilon_1 = 15\nu_e \int_0^\infty k^2 \phi(k) dk,$$

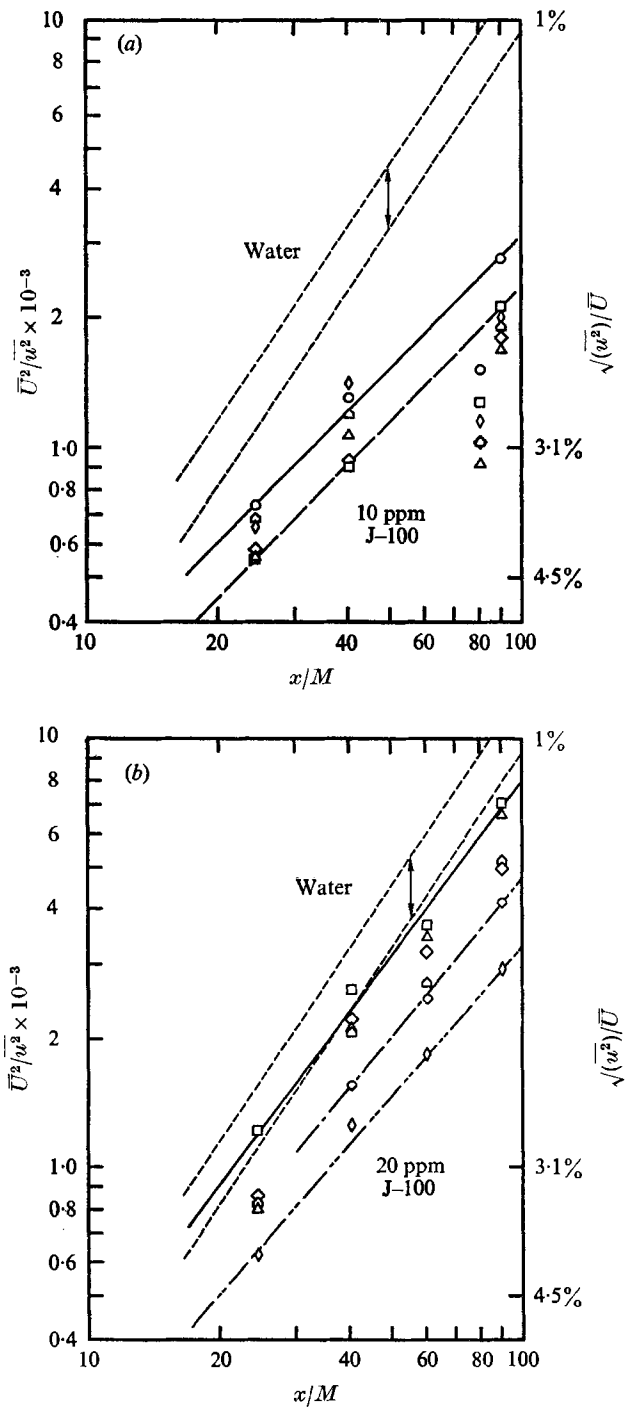


FIGURE 7(a), (b). For legend see p. 187.

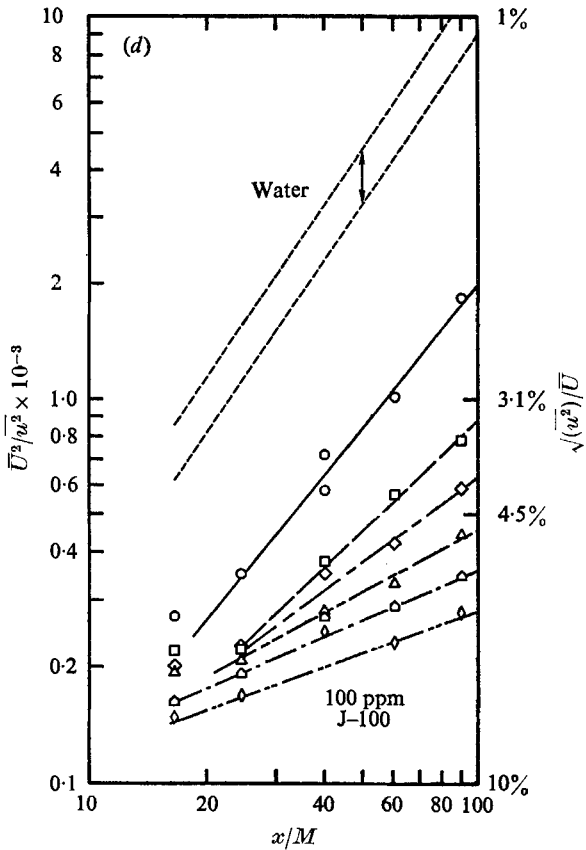
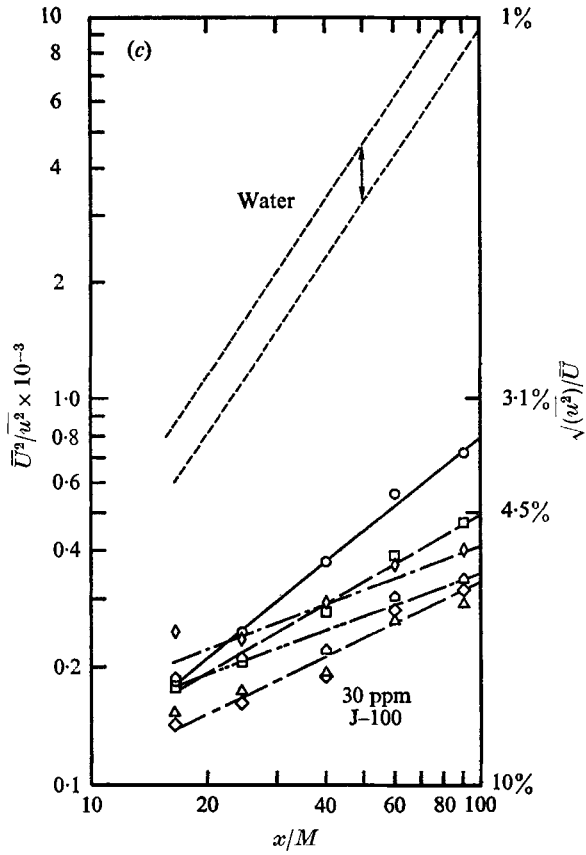


FIGURE 7(c), (d). For legend see p. 187.

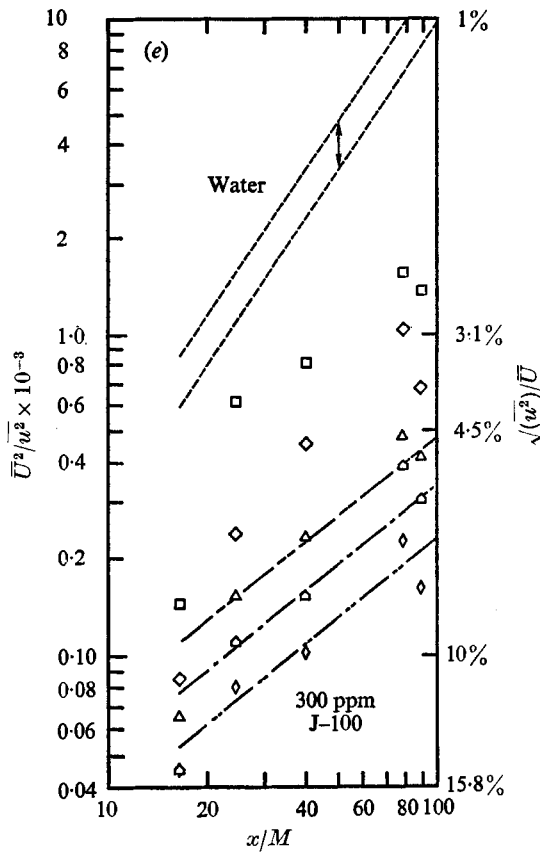


FIGURE 7. Variation of $\overline{U^2}/\overline{u^2}$ with x/M in the polymer solutions, compared to water data (---).

(a) 10 ppm J-100		(b) 20 ppm J-100		(c) 30 ppm J-100	
\overline{U} , cm/sec	n_1	\overline{U} , cm/sec	n_1	\overline{U} , cm/sec	n_1
○ 64	1.0	□ 71	1.34	○ 40	0.82
□ 92	1.0	◇ 114	—	□ 89	0.58
◇ 129	1.0	△ 148	—	◇ 123	0.50
△ 160	—	◻ 178	—	△ 155	0.50
◻ 194	—	▽ 199	—	◻ 186	0.36
◇ 232	—	◇ 223	1.13	◇ 224	0.36

(d) 100 ppm J-100		(e) 100 ppm J-100	
\overline{U} , cm/sec	n_1	\overline{U} , cm/sec	n_1
○ 68	1.25	□ 76	—
□ 108	1.0	◇ 91	—
◇ 140	0.77	△ 120	0.81
△ 168	0.56	◻ 130	0.81
◻ 188	0.44	◇ 150	0.81
◇ 220	0.36		

where ϵ_1 is the viscous dissipation. The viscous dissipation was calculated from the spectral data in the limited inertial subrange, viz. $\phi(k) = \alpha_1 \epsilon_1^{\frac{2}{3}} k^{-\frac{5}{3}}$, where $\alpha_1 = 0.42$, with the assumption that the inertial subrange was unchanged in the polymer solutions. (It should be noted that the use of the measured rates of decay ϵ , to calculate effective viscosities, resulted in values less than that of the solvent.)

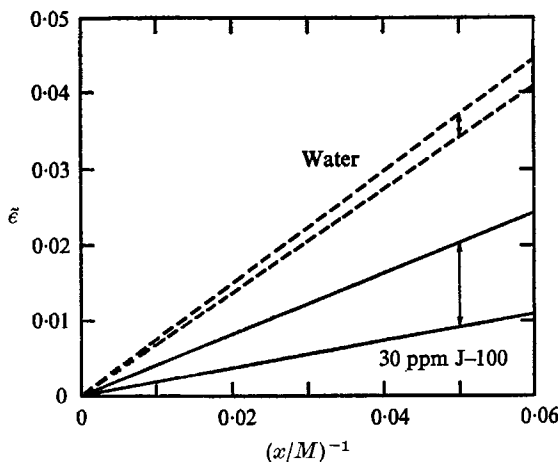


FIGURE 8. Dimensionless decay for 30 ppm J-100 compared to water results.

With ϵ_1 and ν_1 , modified Kolmogoroff variables were formed to make the spectra dimensionless:

$$\tilde{\phi}_e(\tilde{k}_e) = \frac{\phi(k)}{\nu_e^2 k_K^e}, \quad \tilde{k}_e = k/k_K^e, \quad k_K^e \equiv (\epsilon_1/\nu_e^3)^{\frac{1}{4}}.$$

The spectra, non-dimensionalized in this manner, are shown in figures 9(a)-(c), together with the normalized Newtonian spectral data of figure 5 (for which $\nu = \nu_e$ and $\epsilon = \epsilon_1$). The spectral data, for each solution, collapse on to each other when correlated with the modified variables; and, further, they coincide with the universal curve for Newtonian fluids. It appears, then, that there are no differences between the one-dimensional energy spectra in water and these polymer solutions.

A measure of the root-mean-square (r.m.s.) strain rate of the turbulent field is given by

$$\left[\int_0^\infty k^2 \phi(k) dk \right]^{\frac{1}{2}},$$

and viscosity values $\bar{\nu}$ at these r.m.s. strain rates, for each spectrum, were determined from the rheological data. Table 1 shows the comparisons of $\bar{\nu}$ and ν_e , and, in general, the agreement of the two values is satisfactory: they are approximately equal, and both increase with increasing polymer concentration. There are differences between $\bar{\nu}$ and ν_e for the two runs at each concentration; but these may be due to degradation effects, since, in every case, the lower values were for the solutions that had been used longest in the tunnel. Also shown in table 1 are the interesting results that $\epsilon < \epsilon_1$; the rates of decay are less than the viscous

dissipation in the polymer solutions, whereas, for Newtonian fluids, $\epsilon = \epsilon_1$. This implies that there are additional non-Newtonian terms which arise in the energy budget equation for homogeneous, isotropic turbulence of these polymer solutions.

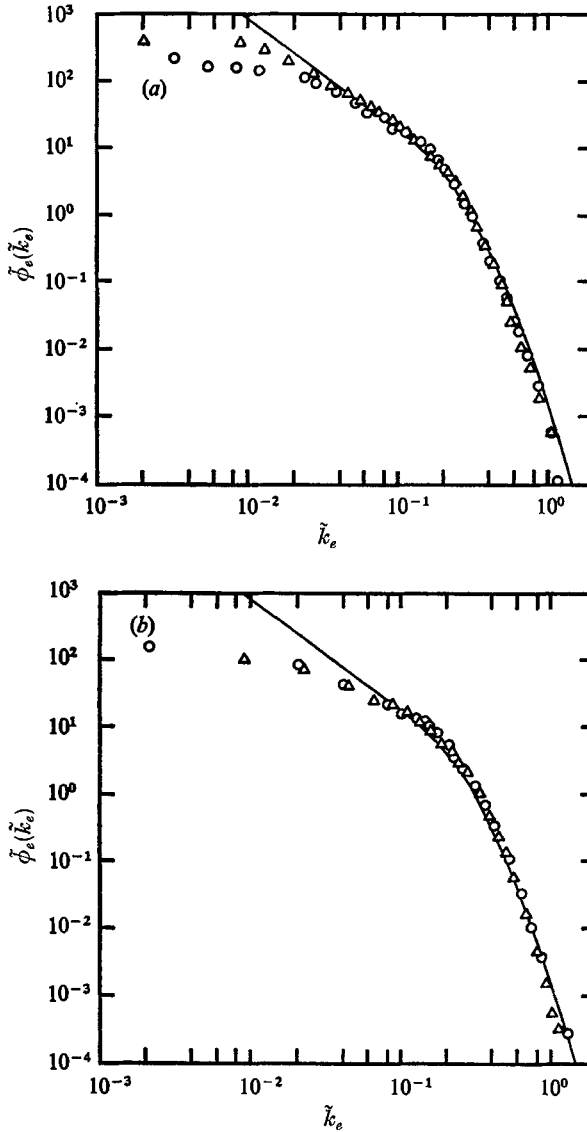


FIGURE 9(a), (b). For legend see p. 190.

4. Discussion and conclusions

The reference measurements of grid pressure drops, streamwise intensities, and one-dimensional energy spectra in water showed, in general, good agreement with previous results for Newtonian fluids. In fact, the precision of the turbulence measurements with the quartz-coated hot-film anemometer probes was only

slightly less than that of those obtained in air with conventional hot-wires, with the exception of the slight attenuation of the spectral signals at high frequencies (high wave-numbers). The dimensionless decay correlation that was introduced

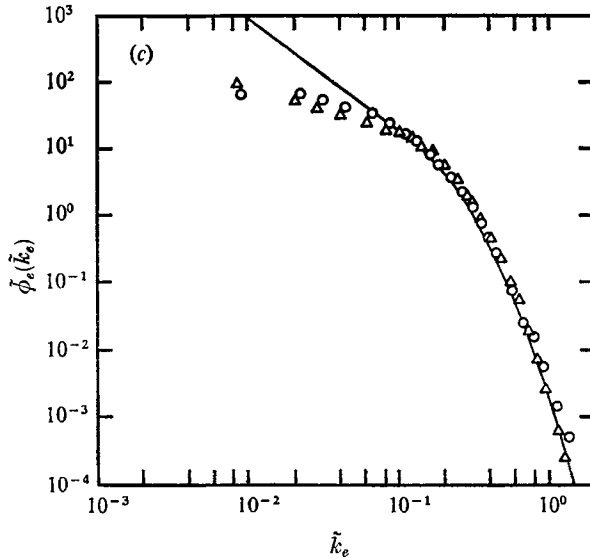


FIGURE 9. Normalized one-dimensional energy spectra in polymer solutions, compared to curve (—) of Gibson & Schwarz (1963) for Newtonian fluids.

(a) 30 ppm J-100, $\bar{U} = 156$ cm/sec ○ $x/M = 16.5$ △ $x/M = 90.5$	(b) 100 ppm J-100, $\bar{U} = 190$ cm/sec ○ $x/M = 16.5$ △ $x/M = 60.5$	(c) 300 ppm J-100, $\bar{U} = 132$ cm/sec ○ $x/M = 24.5$ △ $x/M = 80.5$
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Concentration (ppm)	x/M	ϵ_1 (cm ² /sec ³)	ϵ (cm ² /sec ³)	ν_e (centipoise)	$\bar{\nu}$	r.m.s strain rate (sec ⁻¹)
30	16.5	10,500	970	2.0	1.0†	183
	90.5	2,020	76	1.0	1.0†	114
100	16.5	25,100	1,290	3.5	1.8‡	222
	60.5	10,700	200	2.9	1.9‡	158
300	24.5	8,700	855	4.2	6.0‡	117
	80.5	2,340	100	2.6	7.0‡	78

† Estimated. ‡ From rheological data.

TABLE 1. Comparisons of the effective viscosities ν_e , and the rheological viscosities $\bar{\nu}$ at the observed root-mean-square strain rates; and the rates of decay ϵ , and the viscous dissipation values ϵ_1

(equation (2), figure 4) adequately removed the dependence of the rates of decay on the initial conditions (grid geometry and grid Reynolds number) of grid-generated turbulence for Newtonian fluids, to provide a basis for comparison to the polymer solution decay results.

The grid pressure drop and intensity measurements in the polymer solutions demonstrated that the addition of small quantities of polymer produced significant differences in grid-generated turbulence. The reduction of the pressure drop across the grid and the increased intensity levels indicated that the initial generation of the turbulent field at the grid is markedly different from that in Newtonian fluids. As mentioned earlier, increased intensity levels have been measured in the central core region in the turbulent pipe flow of drag-reducing polymer solutions, and, although not exactly comparable, the present results also show increased turbulence intensities. The results, however, appear contrary to those of Barnard & Sellin (1969), in which decreased bi-normal intensities ($\overline{v^2}$) were observed with the injection of polyethylene oxide into a grid-generated turbulent field. In regard to the reduction of the grid pressure drops, Bate (1967) has observed a somewhat similar phenomenon of a decrease of the pressure drop across an orifice in a turbulent pipe flow system using polyethylene oxide.

The dependence of the streamwise intensities on the distance from the grid was markedly changed in the polymer solutions: the power law exponents were lower than in water. Calculation of the rates of decay in the solutions showed increased values over the x/M range of 16.5 to 100 compared to those in water. Comparison of the dimensionless rates of decay to those in water definitely confirmed that the decay process is different in the polymer solutions from that in water.

The one-dimensional spectral measurements showed no qualitative differences from those in water. A method of normalizing the spectral data was introduced which, although perhaps not unique, adequately correlated all of the spectral data for different polymer concentrations and differing rates of decay on to a single curve, which coincided with that for Newtonian fluids. The spectral normalization method was essentially based on Newtonian turbulence concepts. Lumley's (1964) idea of an effective viscosity for non-Newtonian turbulence was used, but the values were calculated from the equation for Newtonian fluids (which relates the viscous dissipation to the second moment of the spectrum) with the Newtonian kinematic viscosity simply replaced by the effective kinematic viscosity. The viscous dissipation was determined from the spectral data in the inertial subrange, with the assumption that the energy in this range of wavenumbers was unchanged by the addition of polymer. As mentioned above, the use of these parameters to form modified Kolmogoroff variables resulted in good agreement of the spectral data with the Newtonian spectral correlation, which indicates that one-dimensional energy spectra are not altered in these drag-reducing fluids. Further, the effective viscosity values were slightly greater than that of the pure solvent, increased with increasing polymer concentration and were approximately the same as the values determined from the rheological data at the root-mean-square strain rates of the turbulent field. The latter result verified Lumley's concept of an effective viscosity dependent on the r.m.s. strain rate for non-Newtonian turbulence in fluids with a shear dependent viscosity.

The result, that the rate of decay ϵ was less than the viscous dissipation ϵ_1 for the polymer solutions, qualitatively confirms the theoretical predictions of Singh (1966) and Chow & Saibel (1967). However, the exact form of the energy

budget equation for these polymer solutions in homogeneous, isotropic turbulence is not known since the solutions were neither second-order nor Oldroyd fluids.

Of course, the results of the present study have not led to a complete understanding of the causes of drag reduction in turbulent shear flows, but several interesting results were observed in the grid-generated flow, which may indicate those areas to study in shear flows. It appears that the higher-order statistics of the turbulent field, as measured by the energy spectra in these experiments, were not altered by the addition of polymer, but the lower-order measures of grid pressure drops, intensity levels and in particular, the energy budget, were significantly changed in the polymer solutions.

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Note added in proof. Greated (1969) has recently used a laser velocimeter to measure the instantaneous streamwise velocity behind a grid in an 8 ppm polyacrylamide solution. The results showed a decrease of approximately 10% in the root-mean-square intensity compared to water. The experiment was performed in a small tunnel at a low grid Reynolds' number of 1000, with a high solidity ($\sigma = 0.44$) grid. The reference measurements in water were not representative of grid-generated turbulence; intensity levels were twice those usually measured and there were wall effects at distances greater than $25M$ downstream. Spectra, normalized with local intensity, were reported for water and the polymer solution. Near the grid there was some attenuation in the high frequency in the polymer solution, while at $34M$ there was no difference between the spectra in the water and polymer solution.

Clearly, further comparison under similar conditions are required between the different velocity measuring techniques—flow visualization laser velocimeter and hot-film anemometer—to determine their performance in dilute polymer solution.